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The current transport characteristics of a delayed feedback ratchet in a double-well potential

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Abstract

On the basis of the double-well potential which can be calculated theoretically and implemented experimentally, the influence of the time delay, number of particles and asymmetric parameter of the potential on the performance of a delayed feedback ratchet is investigated. The center-of-mass velocity of Brownian particles, average effective diffusion coefficient and *Pe* number are calculated. It is expounded that the parameters are affected by not only the time delay and number of particles but also by the asymmetric parameter of the double-well ratchet potential. It is very interesting to find that the current transport reversal may be obtained by varying the number of particles of the system. It is expected that the results obtained here may be observed in some physical and biological systems because the double-well ratchet potential is realizable experimentally.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Noise introduced transport by Brownian ratchets has attracted many researchers' attention because of its biological interest as well as its potential technological applications [1–4]. For example, flashing ratchets can rectify the thermal motion of diffusive particles by exposing them to a time-dependent, spatially periodic and asymmetric potential [1, 5, 6].

Theoretical and numerical researches of flashing ratchets have revealed a wide range of rich behavior. The following are a few examples: two harmonically coupled particles in an overdamped, flashing ratchet display higher center-of-mass velocity and energetic efficiency than a single particle [7]. Rigid rods of particles in an overdamped, flashing ratchet system display current reversals as a function of size and temperature [8]. A computational model of

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a polyelectrolyte in a flashing ratchet potential demonstrates that the flexibility of a molecular Brownian motor can increase its stall force [9]. Large collections of coupled particles in a flashing ratchet can undergo spontaneous oscillations [10], similar to the cooperative motion of muscle myosin. In addition, one can easily find some overviews on Brownian ratchets and related topics [11].

Most of the Brownian ratchets considered until now use the instant information to operate, that is, they all measure the state of the system and act *instantaneously* according to that measurement. However, in realistic experiment devices there is always a time delay between the input measurement and the output control action due to physical limitations to the velocity of transmission and processing of the information [12, 13]. For example, in the construction of the feedback controlled version of the ratchet in [14], time delays in the feedback will be present due to the finite time needed to take a picture with a CCD camera, transmit it, process it and implement the resulting decision of switching on or off the potential. Feedback flashing ratchets have been recently suggested as a mechanism to explain the stepping motion of the two-headed kinesin [15]. The topic of delay controlled transport has been also addressed by some authors [16, 17]. In another context, a feedback scheme has been used to perform the control of chaotic trajectories in inertia ratchets [18]. Thus, it is necessary to consider and calculate the effects of time delays in the feedback ratchet.

At present the transport reversal in virtue of the variation of the system parameters is a well-known phenomenon in Brownian motors that can be produced by varying the characteristics of the non-equilibrium fluctuations [19] or the parameters of the time-dependent perturbation that drives the system out of equilibrium [20, 21]. However, in most studies of flashing ratchets, the piecewise linear sawtooth potential was used, which is one of the simplest realizations of Brownian motors in general [1]. In the present paper we will adopt the double-well potential [22–25] which can be implemented experimentally to investigate how the structure of the potential affects the current transportation of Brownian particles in the control of delayed feedback. The model presented here is clearly different from those adopted in [16, 17]. We will study the impact of time delay on the effectiveness of the feedback control strategy introduced in [26], so that one can understand more easily how information can be used to improve the performance of the delayed feedback system. It is found that the current reversal of Brownian particles in the double-well ratchet potential can be achieved by varying the number of particles of the system.

2. A delayed feedback ratchet

The feedback ratchet considered here consists of N Brownian particles at temperature T_0 in a periodic potential $U(x)$. One can choose different periodic potentials according to the different aims. Now, the double-well ratchet potential [22–25]

$$U(x) = -U_1\,\mathrm{e}^{-\sin^2(\pi x)/2\sin^2(\pi R)} - U_2\,\mathrm{e}^{-\sin^2(\pi(x-d))/2\sin^2(\pi R)} \tag{1}$$

is used to replace the piecewise linear sawtooth potential used often in literature and its schematic diagram is shown in figure 1, where U_1 and U_2 determine the depth of the stronger and weaker wells, respectively, which are separated by a distance $d = 0.36$ and have width $R = 0.15$, the asymmetry of the potential is $\beta = U_1/U_2 = 10/9$, x is the position, and the ratchet period $L = 1$. The parameters U_1, U_2, d, R, x and L have the same length unit and the ratio of one of them to another is always a dimensionless parameter. The force acting on the particles is $F(x) = -U'(x)$, where the prime denotes the spatial derivative. The state

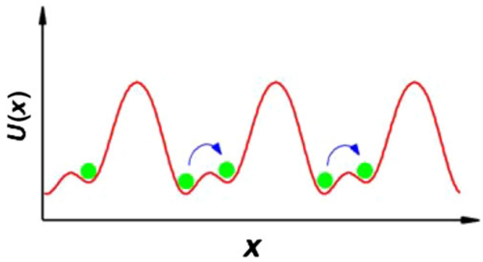

Figure 1. The schematic diagram of the double-well ratchet potential $U(x)$.

of this system is described by the positions $x_i(t)$ of the particles satisfying the overdamped Langevin equations

$$\gamma \dot{x}_i(t) = \alpha(t) F(x_i(t)) + \xi_i(t) \tag{2}$$

and the average force acting on each particle is

$$f(t) = \frac{1}{N} \sum_{i=1}^{N} F(x_i(t)), \tag{3}$$

where γ is the friction coefficient (related to the diffusion coefficient D through Einstein's relation $D = k_B T_0/\gamma$), $\xi_i(t)$ are Gaussian white noises of zero mean and variance $\langle \xi_i(t)\xi_j(t')\rangle = 2\gamma k_B T_0 \delta_{ij}\delta(t-t')$, and $\alpha(t)$ stands for the action of the controller. There is a controller in the system, which measures the sign of the average force and, after a time τ, switches the potential on ($\alpha = 1$) if the ensemble average of the force is positive or switches the potential off ($\alpha = 0$) if it is negative. Thus, the delayed control protocol considered here may be expressed as

$$\alpha(t) = \begin{cases} \Theta(f(t-\tau)), & t \geqslant \tau \\ 0, & \text{otherwise}, \end{cases} \tag{4}$$

where Θ is the Heaviside function [$\Theta(x) = 1$ if $x > 0$, else $\Theta(x) = 0$].

It is interesting to note that, although there are no explicit mechanical interactions between the particles, the use of information in the control of the system introduces a coupling between the particles, in that the force acting on any particle depends on the positions of the other particles. For this reason, for closed-loop strategies [26–32] the average velocity is dependent on the number of particles in the ensemble, in contrast to open-loop policies [2, 5] which, for non-interacting particles, output the same velocity regardless of the ensemble size [33].

The first basic quantity of interest in the system is the center-of-mass velocity of N Brownian particles, which is given by the relation

$$V_{\rm cm} = \lim_{T\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \int_0^T \dot{x}_i(t)\,{\rm d}t. \tag{5}$$

In order to quantify the transport coherence of the coupled Brownian particles, we introduce the Pe number [34–36], i.e.,

$$Pe = \frac{|V_{\rm cm}|L}{D_{\rm eff}}, \tag{6}$$

where $D_{\rm eff}$ is the average effective diffusion coefficient determined by

$$D_{\rm eff} = \lim_{T\to\infty} \sum_{i=1}^{N} \frac{\langle x_i(t)^2\rangle - \langle x_i(t)\rangle^2}{2TN}. \tag{7}$$

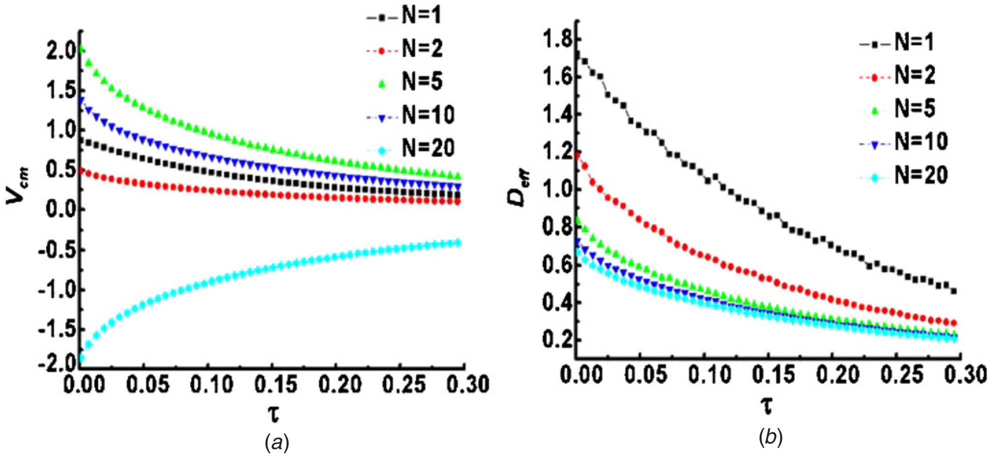

Figure 2. The curves of (*a*) the center-of-mass velocity V_{cm} and (*b*) the average effective diffusion coefficient D_{eff} varying with the time delay τ for different numbers of particles *N*.

The *Pe* number describes the competition between the directional drift and the stochastic diffusion of the particle. The directional drift will increase with the increase of the *Pe* number. Thus, the larger *Pe* number means that the drift predominates over the diffusion and there is high transport coherence.

The numerical solution of equation (2) is performed by using the stochastic Runge–Kutta algorithm. The transport processes of *N* particles are simulated and each trajectory consists of 10^5 steps of integral with the small time step of $h = 10^{-3}$. In the following calculation, the parameters $L = 1$, $d = 0.36$, $R = 0.15$, $U_1 = 3.2$, $\beta = U_1/U_2 = 10/9$, $\gamma = 1$, $D = 1$ and $k_B T_0 = 1$ are chosen, where the unit of energy is the Joule.

3. Results and discussion

In the double-well ratchet potential, we deal with a collective ratchet compounded of a few particles (less than $N < 100$). It will be shown that the center-of-mass velocity V_{cm}, effective diffusion coefficient D_{eff} and *Pe* number are some functions of the various parameters in the system.

3.1. The influence of the delay time

Figure 2(*a*) shows the curves of V_{cm} varying with the delay time τ for different numbers of particles. It is seen that the center-of-mass velocity of the system is a monotonic function of the delay time τ but not a monotonic function of the number of particles *N*. For increasing time delays the correlation between the present sign of the average force and the measured sign that the controller actually uses decreases. It can be understood that the decrease in the center-of-mass velocity is a consequence of the loss of information of the present sign of the average force. Thus, the controller action begins to be uncorrelated to the present state of the system and it effectively begins to act as an open-loop ratchet [30]. The result is compatible with that obtained in [30, 32]. On the other hand, it can be clearly seen that the direction of the net current is closely dependent on the number of particles. We will discuss the phenomenon in detail below.

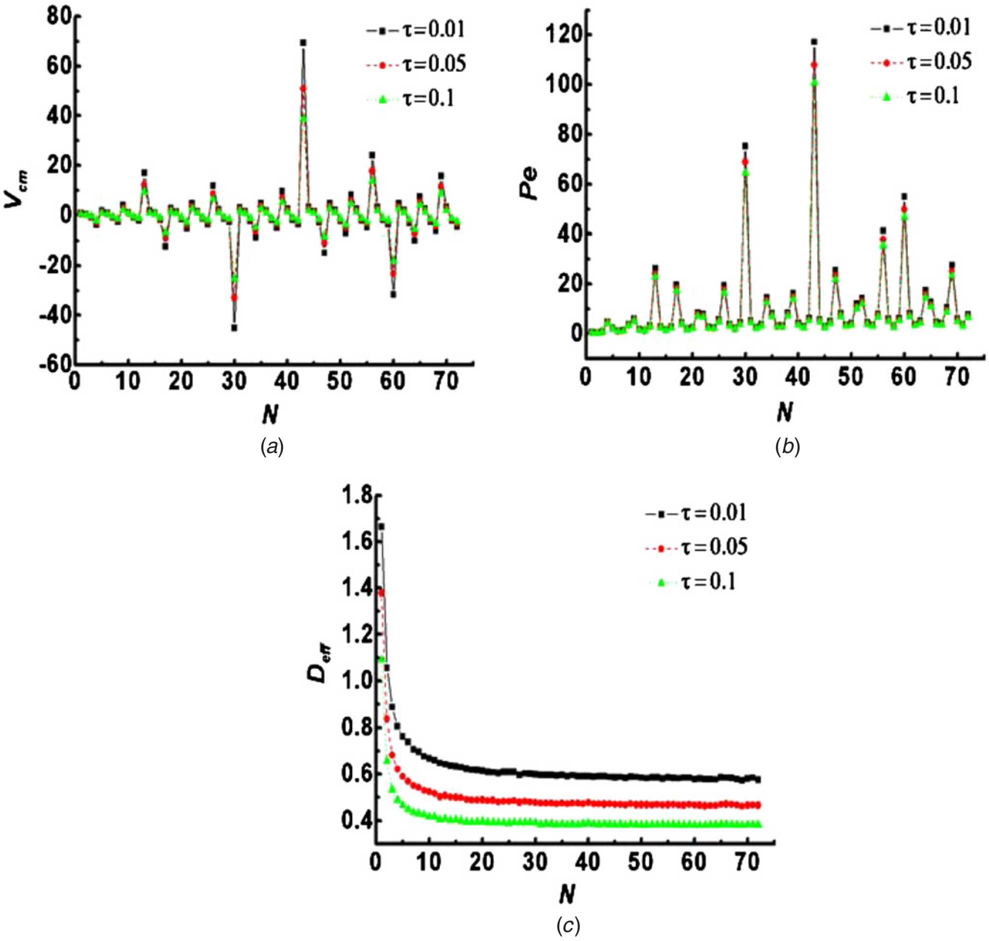

Figure 3. The curves of (*a*) the center-of-mass velocity V_{cm}, (*b*) the *Pe* number and (*c*) the average effective diffusion coefficient D_{eff} varying with the number of particles *N* for different delay times τ.

Figure 2(*b*) shows the effective diffusion coefficient D_{eff} of Brownian particles as a function of the delayed time τ for different numbers of particles. It is easily found that the effective diffusion coefficient D_{eff} of Brownian particles decreases monotonically with the delayed time τ. For the few particle case, particles diffuse more easily, but for the 'many' particle case, it is difficult for the diffusion of particles, which means that the fluctuation of displacement of 'coupled' particles decreases with the increase of particles. Therefore, the average effective diffusion coefficient always decreases with the increase of the number of particles *N*.

3.2. The influence of the number of particles

Figure 3(*a*) shows the center-of-mass velocity as a function of *N* for different delay times τ. It is interesting to find multiple current reversals as *N* increases and the complicated dynamics that depends on the number of particles. The center-of-mass velocity evolves

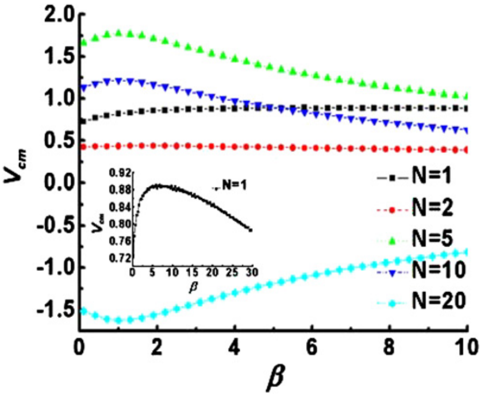

Figure 4. The center-of-mass velocity $V_{\rm cm}$ as a function of asymmetry of the potential β for different N.

quasi-periodically and the current may reverse during one reversal 'period'. But the evolution is not strictly periodic due to the stochastic nature of the dynamics. As analyzed in section 2, the force acting on any particle depends on the positions of the other particles and the average velocity for the closed-loop strategies is dependent on the number of particles in the ensemble. According to figure 2(*a*), when the delay time is zero, the current reversal is found between $N = 10$ and 20. The critical number of particles at the current reversal may be obtained through further calculation. It means that the number of particles can lead to the current reversal. The complex motors can be considered to be the polymer of large biological molecules. A relatively small change in structure such as the change of N can produce the reversal of walking direction. This result is compatible with that obtained in [8]. Therefore, the physical mechanism that leads to the current reversal results from the number of particles N and the structure of the double-well ratchet potential [22].

In order to understand the phenomenon deeply, we calculate the *Pe* number as a function of N for different delay times, as shown in figure 3(*b*). It can be found from figure 3 that when the number of particles is large, the *Pe* number is directly proportional to the absolute value of $V_{\rm cm}$ since $D_{\rm eff}$ may be considered to be constant for a fixed delay. The results show that the *Pe* number can obtain a maximal value during one reversal 'period', which means the 'coupling' between the particles can cause high transport coherence. It is also found that the maximal value of the *Pe* number depends on the maximal velocity of particles in one reversal period.

Figure 3(*c*) shows that for the few particle case, the effective diffusion coefficient $D_{\rm eff}$ decreases quickly when the number of particles is increased; while the number of particles is large, the effective diffusion coefficient $D_{\rm eff}$ does not vary obviously with the increase of N. It indicates once again that the larger the number of particles is, the more difficult the diffusion of particles, the smaller the effective diffusion coefficient, and the slower the decrease speed of the effective diffusion coefficient.

3.3. The influence of the asymmetry parameter

Figure 4 shows the dependence of the center-of-mass velocity on the asymmetry of the potential β for some given values of the number of particles. It is clearly seen that $V_{\rm cm}$ is not a monotonic function of β so that there is an optimized value of β at which the center-of-mass velocity attains its maximum. When the asymmetry of the potential β is large, the center-of-mass velocity decreases with the increase of β. It is seen that the current enhancement may be obtained by changing the structure of the potential. It is found that the number of particles can affect not only the magnitude but also the direction of the current.

4. Conclusions

We have systematically investigated the performance of a feedback ratchet consisting of N Brownian particles confined in the double-well ratchet potential and discussed in detail the influence of the time delay, number of particles and asymmetric parameter of the potential on the center-of-mass velocity of Brownian particles, average effective diffusion coefficient and *Pe* number of a delayed feedback ratchet. It is very interesting to find that the current transport reversal may be achieved by varying the number of particles of the system and that the current enhancement may be obtained by changing the structure of the potential. The theoretical results obtained here are significant because they may be observed in some physical and biological systems by using the experimentally realizable double-well ratchet potential.

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